Three design factors in this experiment:

|  |  |  |  |
| --- | --- | --- | --- |
| code | paper height(mm) | width-height ratio | leg length(mm) |
| -1 | 148.5 | 0.4 | 8 |
| 1 | 210 | 0.5 | 10 |



Unwanted noises:

1. The material property of papers may vary in each frog.
2. The humidity in air will affect the elasticity of paper.
3. The angle between the finger and a frog will vary in each jumping test.
4. The friction coefficient between frog and finger may change because of sweat on hand.
5. The finger’s force and placement on a frog will vary in each jumping test.

* X1: paper height
* X2: width-height ratio
* X3: leg length

factorial experiment:

|  |  |  |  |
| --- | --- | --- | --- |
| test | x1 | x2 | x3 |
| 1 | -1 | -1 | -1 |
| 2 | 1 | -1 | -1 |
| 3 | -1 | 1 | -1 |
| 4 | 1 | 1 | -1 |
| 5 | -1 | -1 | 1 |
| 6 | 1 | -1 | 1 |
| 7 | -1 | 1 | 1 |
| 8 | 1 | 1 | 1 |

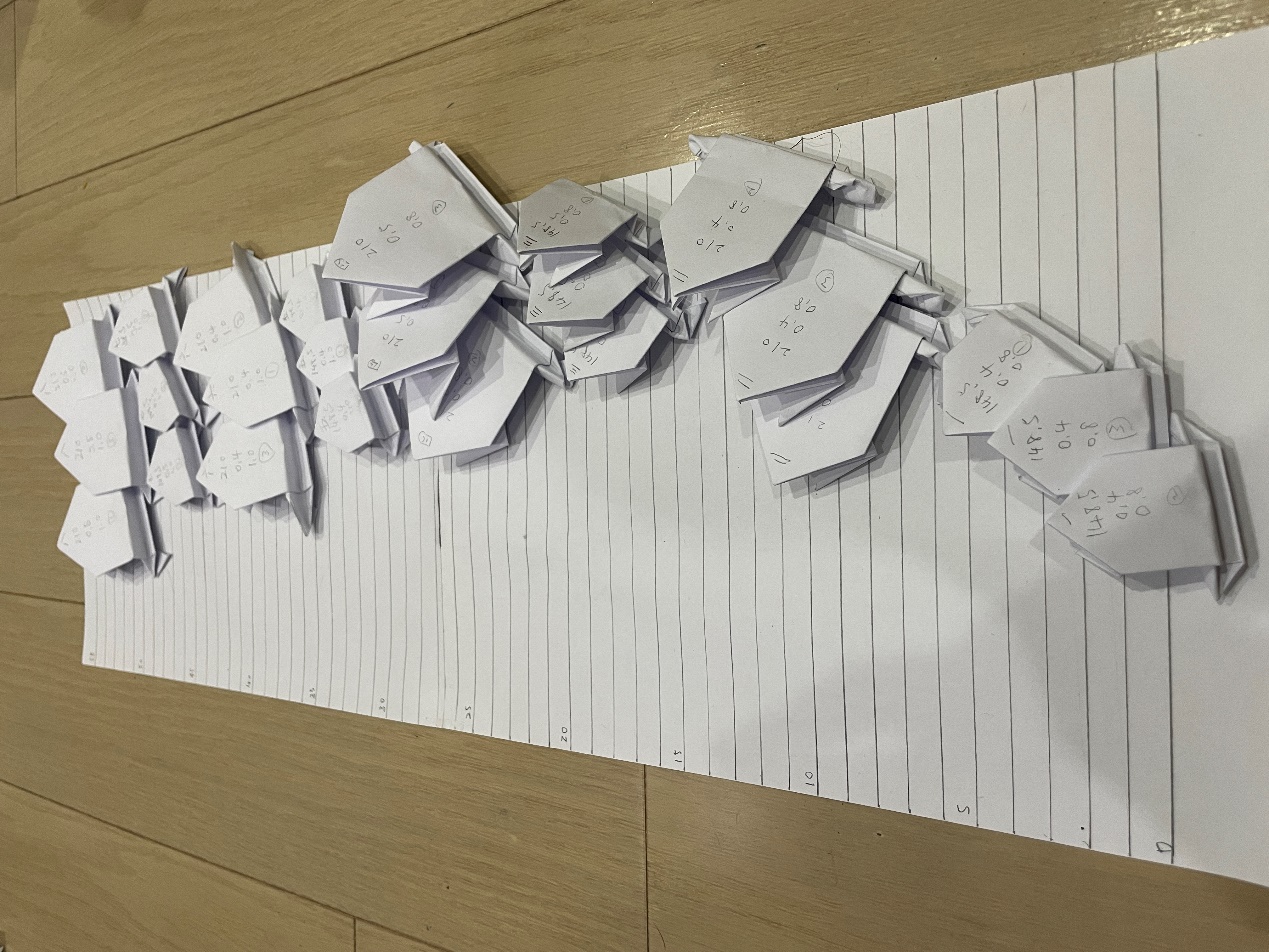
In my experiment, I will create three identical frogs for each combination of factors. To ensure randomization, I will randomize the order of the jumping trials instead of following a fixed sequence for each run. Since there are three runs, each frog will perform three jumps.

Experiment table:

How to handle the noises:

* To prevent I using my fingernails to press paper frogs, I always cut my nails before I do experiment.
* Do more replication and randomize the experimental trials to make noises more uniformly and more randomly distributed.







Experiment details:

* Day: 6/10 Monday
* time: 11:00am~12:30pm
* place: home
* witness: 王邑宇 (reason: he is my brother)

measurement method:

* Measurement plate: Design a plate with markings ranging from 0 to 55 cm (as the picture in 4.)
* Align frogs: Position each paper frog so that its rear edge aligns with the zero line on the measurement plate before each jump.
* Execute a jump: Use fingertip to press down on the back of the frog and release it to initiate the jump
* Valid jumps: If the frog rolls or bounces twice or more, disregard the result and perform the jump again.
* Distance measurement: For each valid jump, measure the shortest distance between the zero line and the landing point of the frog. This distance is the result for that trial.

experiment result:



Parameters of this experiment:

Average and sample variance in each test:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | replicate 1 | | | replicate 2 | | | replicate 3 | | |  |  |
| test | x1 | x2 | x3 | run1 | run2 | run3 | run1 | run2 | run3 | run1 | run2 | run3 | y\_bar | s\_i^2 |
| 1 | -1 | -1 | -1 | 255 | 300 | 285 | 240 | 310 | 300 | 280 | 300 | 360 | 292.2222 | 1175.694 |
| 2 | 1 | -1 | -1 | 200 | 205 | 230 | 220 | 155 | 200 | 150 | 165 | 200 | 191.6667 | 806.25 |
| 3 | -1 | 1 | -1 | 150 | 175 | 160 | 190 | 150 | 190 | 220 | 195 | 205 | 181.6667 | 606.25 |
| 4 | 1 | 1 | -1 | 170 | 210 | 160 | 160 | 170 | 185 | 150 | 155 | 200 | 173.3333 | 431.25 |
| 5 | -1 | -1 | 1 | 300 | 290 | 345 | 230 | 285 | 270 | 275 | 295 | 385 | 297.2222 | 1994.444 |
| 6 | 1 | -1 | 1 | 140 | 150 | 145 | 200 | 150 | 185 | 145 | 140 | 135 | 154.4444 | 502.7778 |
| 7 | -1 | 1 | 1 | 175 | 210 | 230 | 230 | 210 | 245 | 245 | 210 | 275 | 225.5556 | 815.2778 |
| 8 | 1 | 1 | 1 | 180 | 175 | 205 | 200 | 250 | 205 | 200 | 175 | 205 | 199.4444 | 527.7778 |

Effect coefficient:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| E1 | E2 | E3 | E12 | E23 | E13 | E123 |
| -69.4444 | -38.8889 | 9.444444 | 52.22222 | 25.55556 | -15 | 6.111111111 |

Assuming all effects are null effect:

Degree of freedom for t-test:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | t-test | p-value | t\_64,0.025 | t\_64,0.975 |
| E1 | -69.4444 | -10.0616 | 8.28E-15 | -1.99773 | 1.997729654 |
| E2 | -38.8889 | -5.63448 | 4.23E-07 | -1.99773 | 1.997729654 |
| E3 | 9.444444 | 1.368373 | 0.17598 | -1.99773 | 1.997729654 |
| E12 | 52.22222 | 7.566296 | 1.88E-10 | -1.99773 | 1.997729654 |
| E23 | 25.55556 | 3.702656 | 0.000446 | -1.99773 | 1.997729654 |
| E13 | -15 | -2.1733 | 0.033465 | -1.99773 | 1.997729654 |
| E123 | 6.111111 | 0.885418 | 0.379246 | -1.99773 | 1.997729654 |

reject , they are significant factors.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| source of variation | SS | DOF | MS |  |  |
| E1 | 86805.56 | 1 | 86805.56 | 100.2381 | 3.986269 |
| E2 | 27222.22 | 1 | 27222.22 | 31.43468 | 3.986269 |
| E12 | 49088.89 | 1 | 49088.89 | 56.68507 | 3.986269 |
| E23 | 11755.56 | 1 | 11755.56 | 13.57465 | 3.986269 |
| E13 | 4050 | 1 | 4050 | 4.676711 | 3.986269 |
| error | 57155.56 | 66 | 865.9933 |  |  |
| total | 236077.8 | 71 |  |  |  |



Suppose that residuals are normal distributed. The parameters of normal distribution are and

* Rank each residual from 1~72
* The c.d.f. of effect is:
* The normal distribution value of effect:

Plot the residuals vs. run orders:

Plot the residuals vs.

Plot the residuals vs. :

Plot the residuals vs. x1:

Plot the residuals vs. x2:

Plot the residuals vs. x3:

Bartlett’s test ():

Reject if :



SN ratio: Larger-the-best

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | replicate 1 | | | replicate 2 | | | replicate 3 | | |  |
| test | x1 | x2 | x3 | run1 | run2 | run3 | run1 | run2 | run3 | run1 | run2 | run3 | SN |
| 1 | -1 | -1 | -1 | 255 | 300 | 285 | 240 | 310 | 300 | 280 | 300 | 360 | 49.15775 |
| 2 | 1 | -1 | -1 | 200 | 205 | 230 | 220 | 155 | 200 | 150 | 165 | 200 | 45.37338 |
| 3 | -1 | 1 | -1 | 150 | 175 | 160 | 190 | 150 | 190 | 220 | 195 | 205 | 44.96722 |
| 4 | 1 | 1 | -1 | 170 | 210 | 160 | 160 | 170 | 185 | 150 | 155 | 200 | 44.62473 |
| 5 | -1 | -1 | 1 | 300 | 290 | 345 | 230 | 285 | 270 | 275 | 295 | 385 | 49.21638 |
| 6 | 1 | -1 | 1 | 140 | 150 | 145 | 200 | 150 | 185 | 145 | 140 | 135 | 43.57489 |
| 7 | -1 | 1 | 1 | 175 | 210 | 230 | 230 | 210 | 245 | 245 | 210 | 275 | 46.87 |
| 8 | 1 | 1 | 1 | 180 | 175 | 205 | 200 | 250 | 205 | 200 | 175 | 205 | 45.85851 |

Effect coefficient:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| E1 | E2 | E3 | E12 | E23 | E13 | E123 |
| -2.694963152 | -1.250482833 | 0.349173936 | 2.017970648 | 1.219103481 | -0.631532694 | 0.297030052 |

Assuming all effects are null effects, and estimated standard error by the lowest three effects.

and are relatively small, therefore, they are used for testing the statistical significance of main effects and larger interaction effects.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | effect | t-test | p-value | t\_2,0.025 | t\_2,0.975 |
| E1 | -2.694963152 | -5.461053371 | 0.031934 | -4.30265 | 4.302653 |
| E2 | -1.250482833 | -2.533969152 | 0.126788 | -4.30265 | 4.302653 |
| E3 | 0.349173936 | 0.707563479 | 0.552555 | -4.30265 | 4.302653 |
| E12 | 2.017970648 | 4.089200775 | 0.054923 | -4.30265 | 4.302653 |
| E23 | 1.219103481 | 2.470382266 | 0.132146 | -4.30265 | 4.302653 |
| E13 | -0.631532694 | -1.279733174 | 0.329027 | -4.30265 | 4.302653 |
| E123 | 0.297030052 | 0.601899497 | 0.608386 | -4.30265 | 4.302653 |

It shows that only is significant effect to the SN ratio. Consequently, the predictive model of SN ratio is:



Based on the predictive model of jumping distance of paper frogs, a best combination of three factor can be find:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 |  | x1 | x2 | x3 |  |
| -1 | -1 | -1 | 300 | -1 | -1 | 1 | 289.4444 |
| 1 | -1 | -1 | 193.3333 | 1 | -1 | 1 | 152.7778 |
| -1 | 1 | -1 | 183.3333 | -1 | 1 | 1 | 223.8889 |
| 1 | 1 | -1 | 181.1111 | 1 | 1 | 1 | 191.6667 |

The optimum setting of three factors is

On the other hand, the predictive model of SN ratio is:

|  |  |
| --- | --- |
| x1 |  |
| -1 | 47.55283923 |
| 1 | 44.85787608 |

It shows that when , the jumping distance SN ratio of paper frogs is largest.

As a result, there is no conflict between the predictive model of jumping distance and the predictive model of the SN ratio for jumping distance. The estimated indicates that the SN ratio performs best when . Similarly, the estimated shows that the optimal jumping distance occurs when .



|  |  |  |  |
| --- | --- | --- | --- |
| test | x1 | x2 | x3 |
| 1 | -1 | -1 | -1 |
| 2 | 1 | -1 | -1 |
| 3 | -1 | 1 | -1 |
| 4 | 1 | 1 | -1 |
| 5 | -1 | -1 | 1 |
| 6 | 1 | -1 | 1 |
| 7 | -1 | 1 | 1 |
| 8 | 1 | 1 | 1 |
| 9 | -1.682 | 0 | 0 |
| 10 | 1.682 | 0 | 0 |
| 11 | 0 | -1.682 | 0 |
| 12 | 0 | 1.682 | 0 |
| 13 | 0 | 0 | -1.682 |
| 14 | 0 | 0 | 1.682 |
| 15 | 0 | 0 | 0 |



Experiment details:

* Day: 6/10 Monday
* time: 1:30pm~2:30pm
* place: home
* witness: 王邑宇 (reason: he is my brother)

measurement method: same method in (5)

experiment result:



Parameters of this experiment:

Average and sample variance in each test:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | Replication1 | | | Replication2 | | | Replication3 | | |  |  |
| test | x1 | x2 | x3 | run1 | run2 | run3 | run1 | run2 | run3 | run1 | run2 | run3 | y\_bar | s\_i^2 |
| 1 | -1 | -1 | -1 | 255 | 300 | 285 | 240 | 310 | 300 | 280 | 300 | 360 | 292.2222 | 1175.694 |
| 2 | 1 | -1 | -1 | 200 | 205 | 230 | 220 | 155 | 200 | 150 | 165 | 200 | 191.6667 | 806.25 |
| 3 | -1 | 1 | -1 | 150 | 175 | 160 | 190 | 150 | 190 | 220 | 195 | 205 | 181.6667 | 606.25 |
| 4 | 1 | 1 | -1 | 170 | 210 | 160 | 160 | 170 | 185 | 150 | 155 | 200 | 173.3333 | 431.25 |
| 5 | -1 | -1 | 1 | 300 | 290 | 345 | 230 | 285 | 270 | 275 | 295 | 385 | 297.2222 | 1994.444 |
| 6 | 1 | -1 | 1 | 140 | 150 | 145 | 200 | 150 | 185 | 145 | 140 | 135 | 154.4444 | 502.7778 |
| 7 | -1 | 1 | 1 | 175 | 210 | 230 | 230 | 210 | 245 | 245 | 210 | 275 | 225.5556 | 815.2778 |
| 8 | 1 | 1 | 1 | 180 | 175 | 205 | 200 | 250 | 205 | 200 | 175 | 205 | 199.4444 | 527.7778 |
| 9 | -1.682 | 0 | 0 | 355 | 390 | 300 | 280 | 350 | 350 | 395 | 405 | 355 | 353.3333 | 1750 |
| 10 | 1.682 | 0 | 0 | 220 | 300 | 200 | 240 | 230 | 310 | 225 | 200 | 215 | 237.7778 | 1625.694 |
| 11 | 0 | -1.682 | 0 | 330 | 350 | 410 | 345 | 370 | 410 | 370 | 385 | 400 | 374.4444 | 846.5278 |
| 12 | 0 | 1.682 | 0 | 220 | 215 | 215 | 240 | 230 | 275 | 260 | 285 | 270 | 245.5556 | 752.7778 |
| 13 | 0 | 0 | -1.682 | 220 | 210 | 230 | 255 | 245 | 250 | 250 | 215 | 260 | 237.2222 | 350.6944 |
| 14 | 0 | 0 | 1.682 | 205 | 255 | 250 | 220 | 285 | 255 | 195 | 195 | 220 | 231.1111 | 992.3611 |
| 15 | 0 | 0 | 0 | 220 | 255 | 265 | 215 | 210 | 245 | 235 | 260 | 225 | 236.6667 | 418.75 |

Check the correlation coefficient between each factor (including quadratic terms):

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x1x2 | x2x3 | x1x3 | x1x2x3 | x1^2 | x2^2 | x3^2 | y |
| x1 | 1 |  |  |  |  |  |  |  |  |  |  |
| x2 | 0 | 1 |  |  |  |  |  |  |  |  |  |
| x3 | 0 | 0 | 1 |  |  |  |  |  |  |  |  |
| x1x2 | 0 | 0 | 0 | 1 |  |  |  |  |  |  |  |
| x2x3 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |  |  |
| x1x3 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |  |
| x1x2x3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |
| x1^2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |
| x2^2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.3834 | 1 |  |  |
| x3^2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.3834 | -0.3834 | 1 |  |
| y | -0.48837 | -0.38515 | 0.028444 | 0.282323 | 0.138158 | -0.08109 | 0.033038 | 0.091104 | 0.182951 | -0.29925 | 1 |

The result shows that there is little correlation between each factor, therefore, it can be considered as no multicollinearity effect among this regression models:

Regression analysis results:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |
| R 的倍數 | 0.767981 |  |  |  |  |  |
| R 平方 | 0.589795 |  |  |  |  |  |
| 調整的 R 平方 | 0.556713 |  |  |  |  |  |
| 標準誤 | 45.1373 |  |  |  |  |  |
| 觀察值個數 | 135 |  |  |  |  |  |
|  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |
| 迴歸 | 10 | 363238.7 | 36323.87 | 17.82875 | 9.25E-20 |  |
| 殘差 | 124 | 252634.6 | 2037.376 |  |  |  |
| 總和 | 134 | 615873.3 |  |  |  |  |
|  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% |
| 截距 | 249.6759 | 14.95868 | 16.69103 | 1.67E-33 | 220.0685 | 279.2833 |
| x1 | -34.5683 | 4.071147 | -8.49104 | 5.31E-14 | -42.6262 | -26.5103 |
| x2 | -27.2617 | 4.071147 | -6.69631 | 6.63E-10 | -35.3196 | -19.2037 |
| x3 | 2.013354 | 4.071147 | 0.494542 | 0.621798 | -6.04459 | 10.0713 |
| x1x2 | 26.11111 | 5.319482 | 4.908582 | 2.83E-06 | 15.58237 | 36.63986 |
| x2x3 | 12.77778 | 5.319482 | 2.402072 | 0.017786 | 2.249033 | 23.30652 |
| x1x3 | -7.5 | 5.319482 | -1.40991 | 0.161069 | -18.0287 | 3.028745 |
| x1x2x3 | 3.055556 | 5.319482 | 0.574409 | 0.566732 | -7.47319 | 13.5843 |
| x1^2 | 2.76179 | 6.114933 | 0.451647 | 0.652312 | -9.34138 | 14.86496 |
| x2^2 | 7.867414 | 6.114933 | 1.28659 | 0.200634 | -4.23575 | 19.97058 |
| x3^2 | -18.9371 | 6.114933 | -3.09686 | 0.002419 | -31.0403 | -6.83395 |

First, the table shows that reject the null hypothesis with . Therefore, they are significant effects. Second, look closely on the ANOVA table, the value of F-test is larger than the critical value(“顯著值”). This indicates that the observed differences between tests are not caused by noise. Finally, the R^2 and adjusted R^2 are 0.589795 and 0.556713 respectively. This implies that the proportion of the variation explained by model is more than half.



Based on the first regression analysis table, a predictive regression model of the paper frogs’ jumping distance can be constructed:

Final regression analysis:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |
| R 的倍數 | 0.758411 |  |  |  |  |  |
| R 平方 | 0.575187 |  |  |  |  |  |
| 調整的 R 平方 | 0.558722 |  |  |  |  |  |
| 標準誤 | 45.03493 |  |  |  |  |  |
| 觀察值個數 | 135 |  |  |  |  |  |
|  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |
| 迴歸 | 5 | 354242.6 | 70848.52 | 34.93267 | 1.86E-22 |  |
| 殘差 | 129 | 261630.7 | 2028.145 |  |  |  |
| 總和 | 134 | 615873.3 |  |  |  |  |
|  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% |
| 截距 | 263.065 | 5.582995 | 47.11899 | 3.69E-83 | 252.019 | 274.1111 |
| x1 | -34.5683 | 4.061914 | -8.51034 | 3.8E-14 | -42.6049 | -26.5317 |
| x2 | -27.2617 | 4.061914 | -6.71153 | 5.5E-10 | -35.2983 | -19.2251 |
| x1x2 | 26.11111 | 5.307418 | 4.919739 | 2.59E-06 | 15.61026 | 36.61197 |
| x2x3 | 12.77778 | 5.307418 | 2.407532 | 0.017476 | 2.276922 | 23.27863 |
| x3^2 | -23.0124 | 4.41302 | -5.21466 | 7.13E-07 | -31.7437 | -14.2811 |

The analysis table shows that all effects in the predictive model are significant. The F-test value in ANOVA table is still larger than the critical value, implying that the observed differences between vary factors are statistically significant and not merely due to random noise. The R^2 and adjusted R^2 are 0.575187 and 0.558722, respectively. This indicates that the model explains over half of the variation in the data.

Compare to the predictive model built in (6), several similarities and differences are observed.

Predictive model in (6):

Predictive model in (15):

* Similarity: Both models include five factors in their analysis, and they take as significant effects. This suggests that these main effect and interaction effect play a crucial rule in the jumping distance.
* Difference: In the current model, the term is included as a significant effect. On the other hand, the previous model from (6) considers the interaction term. This indicates that there is a critical nonlinear effect is , whereas the interaction effect of is not crucial in this model.